THE EARTH'S MAGNETOSPHERE

The knowledge we are going to gain here will be applicable to any planetary object with magnetic field facing solar wind.

The Earth's magnetic field can be approximated by a simple dipole (Duffin 1990, Parks 2004)

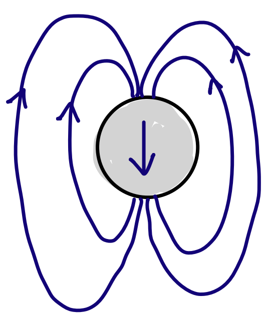
From first approximation (simplest approximation):

$$B_r = -2B_0\left(\frac{R_E}{r}\right)^3 \cos\theta$$

$$B_\theta = -B_0\left(\frac{R_E}{r}\right)^3 \sin\theta$$

$$B_\phi = 0$$

B_0 = Base mag. field
R_E = radius of Earth
r = distance from center.



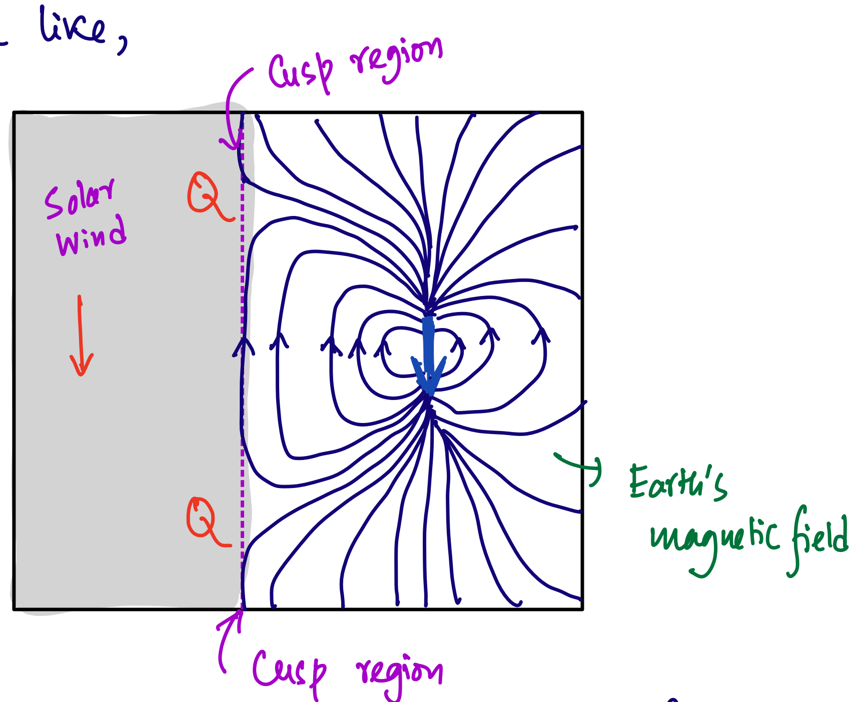
DIPOLE APPROX.

Approximations

Solar wind

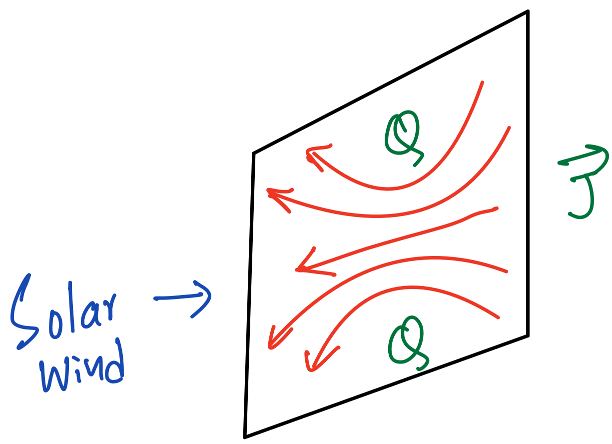
→ a wall of ideally conducting plasma ($\sigma \to \infty$)
→ no B-field (i.e. IMF)
→ FROZEN-IN-FIELD is valid (No Diffusion)

Using the approximations, the interaction between the solar wind and the Earth's magnetic field lines will look like,



The assumption of S.W. bearing no magnetic field will force us to consider surface induced magnetic field in the opposite direction when they interact with the Earth's magnetic field such that solar wind remains the same.

Due to interaction between Solar Wind and the Earth's magnetic field, there will be some change in the Earth's side as well. The field lines will adapt to accomodate the new change and in due process will produce cusp region (a region with open mag. field lines on both sides)

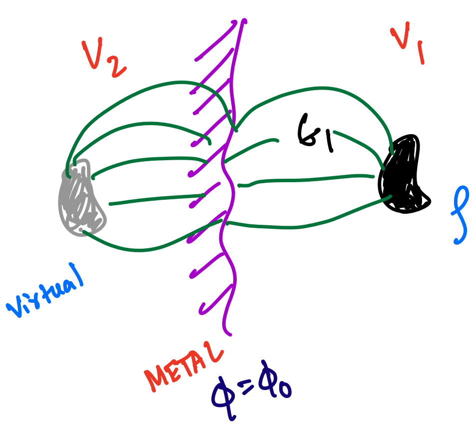


Surface current paths at the interface

METHOD OF IMAGES

This method is quite robust to determine field structure in presence of a sharp boundary.

For our case, the sharp boundary is our solar wind boundary.

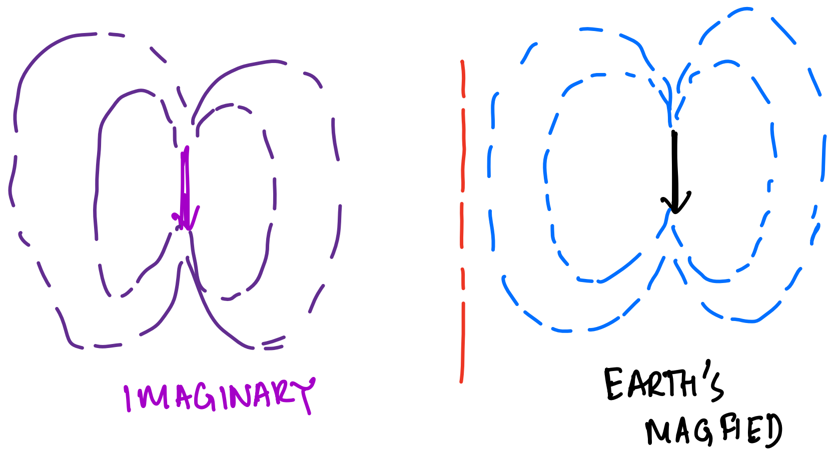


i) Remove the conductor in V_2

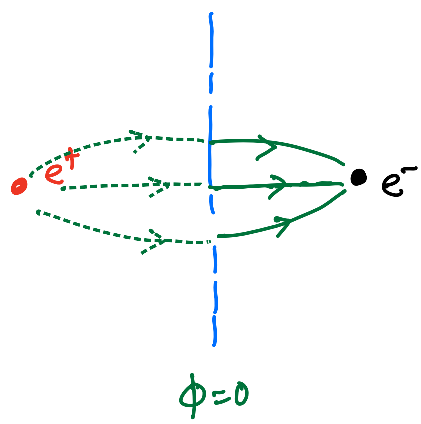
ii) Fill V_2 with same dielectric const.

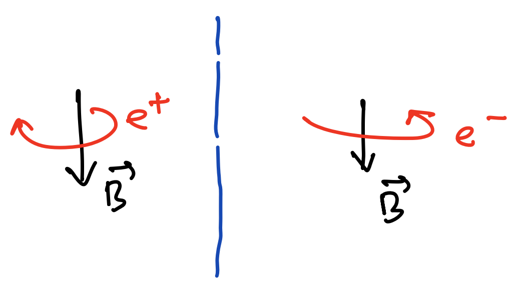
iii) Introduce charges in V_2 such that the potential at the surface and at infinity is fulfilled.

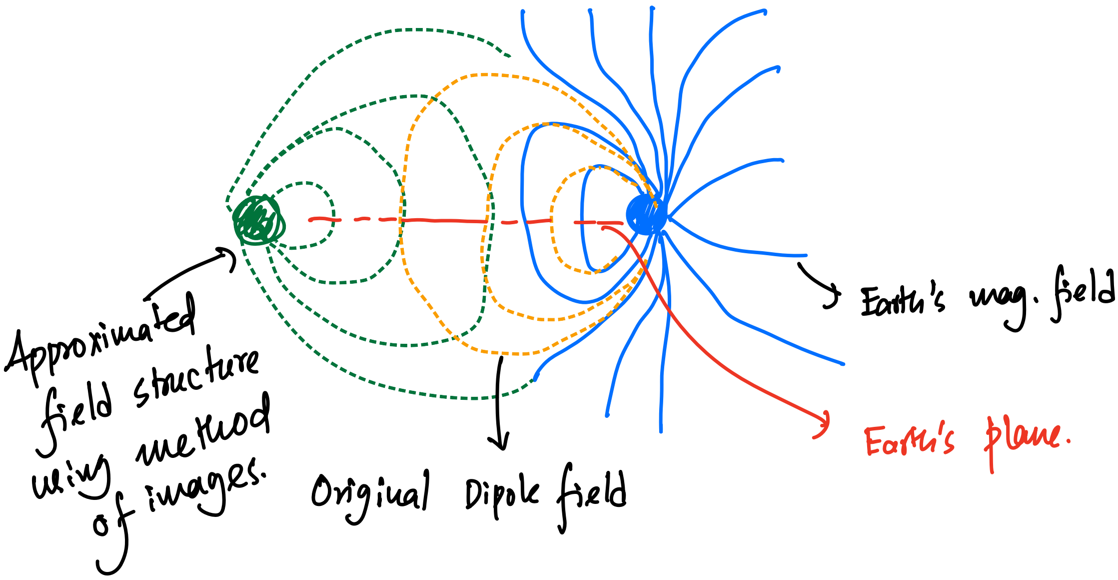
Using the same method to find the currents in the solar wind.



Similar situation can be found for point charge in front of a metalic surface.







In reality, Earth's magnetic field is strongest at the Earth's plane and weakest in the upper region.

So, the strong current will be at the plane and weaker in the upper side.

Now, let's find out the reality. If the assumptions we took for solar wind is valid.

For Solar Wind: $\underline{\sigma} = 5 \cdot 10^6$ S/m

flow velocity $u_0 = 10^5$ m/s

Magnetic Reynold's Number

$$R_L = \mu_0 \underline{\sigma} u_0 L$$

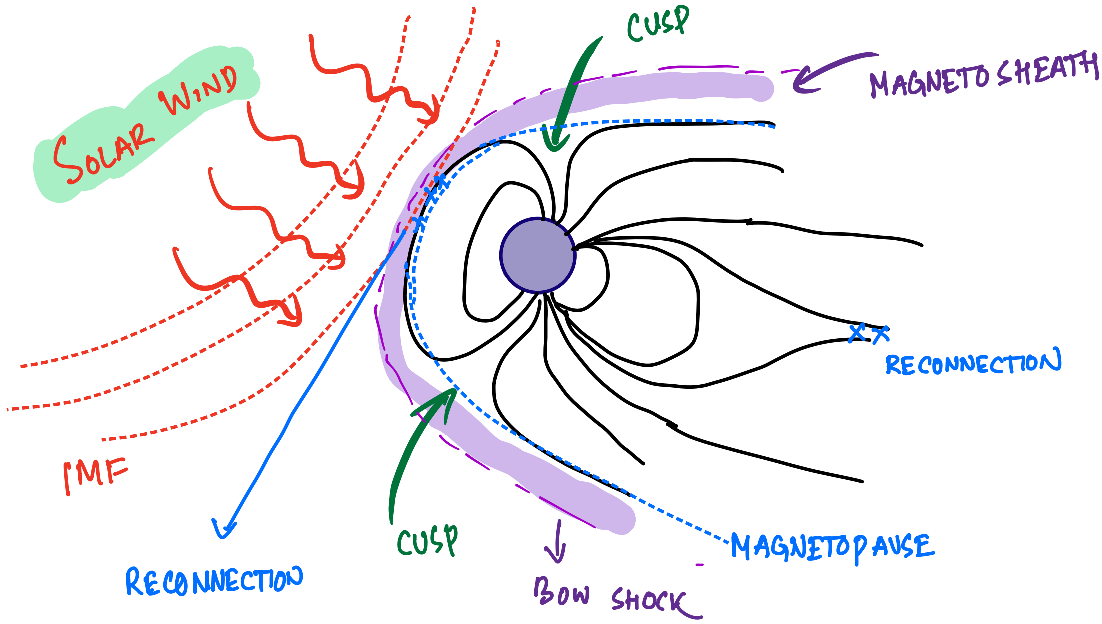
Considering, $L > 10^3$ km.

$$R_L \approx 6 \cdot 10^8 \gg 1$$

Therefore, our assumptions for ideal MHD approximation still holds.

For solar wind, the diffusion is highly unlikely considering the continuous supply of solar winds and large scale length.

REALISTIC VERSION OF MAGNETOSPHERE



When solar wind carries the magnetic field, it will interact with the Earth's magnetosphere. The points where it physically interacts (very small scale), the diffusive term comes into picture (so as the resistivity).

Such coupling between the Earth's magnetic field and the interplanetary magnetic field (IMF) is called magnetic reconnection.

Reconnections first appear in the day-side but also extends to the night-side.

Reconnection events breaks the frozen in field condition locally, which allows the energetic particles of solar wind jumps to the Earth's magnetic field lines and enters the Earth's atmosphere and produce auroras.

THE CURRENT REQUIRED IN THE MAGNETOSHEATH

$\Delta B = 20 nT$

Current in the magnetosheath $K = \frac{\Delta B}{\mu_0} \approx 16 \frac{mA}{m}$

Integrating over 1000 km $= 16 kA$

At BOW SHOCK the pressure due to solar wind and the Earth's magnetosphere is balanced to form magnetosheath.

MAGNETIC PRESSURE : $\frac{B^2}{2\mu_0} \Rightarrow |B| = 2B_0 \left(\frac{R_E}{r}\right)^3 \cos 0°$ (Earth's plane)

$$= 2 B_0 R_E^3 \frac{1}{r^3}$$

We can also express $|B|$ as, $|B| = \frac{2\mu_0 m}{4\pi r^3}$ ← magnetic moment

So, we can write, $B_0 = \frac{\mu_0 m}{4\pi R_E^3}$

SOLAR WIND PRESSURE: $p = nu \cdot mu$ ← momentum received per sec per area.

$= nu^2 M$ ↑ AVG. ION MASS

ignoring thermal spread.

Now, we can write,

$$\frac{B^2}{2\mu_0} = nu^2 M$$

$$\Rightarrow \frac{2\mu_0 m^2}{(4\pi R_p^3)^2} = u^2 n M$$

$$\Rightarrow R_p = \left[\frac{\mu_0 m^2}{8\pi^2 u^2 n M}\right]^{1/6}$$

Using actual values, $u_0 = 10^5 \, m/s$, $n = 5 \cdot 10^6 \, m^{-3}$

$M = 1.66 \cdot 10^{-27} \, kg$

we have, $R_p = 16 \, R_E$, where $R_E \sim 6 \cdot 10^3$ km.

In reality the distance of the magnetopause is ~10-15 R_E. If we consider the tilt angle of Earth, the number will go down more closer. We can also add the thermal pressure.

Remember, SW is a dynamic process. Since $R_p = f(u, n)$, the R_p will keep changing.