DEBYE LENGTH

POISSON'S EQN.

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \phi(\vec{r}) = \frac{e}{\epsilon_0}\left[n_e(\vec{r}) - n_i(\vec{r}) - \frac{q}{e}\,\delta(\vec{r})\right]$$

Assuming, ions are immobile and $n_i = n_0$ (unperturbed density)

The stationary solution for the problem can be derived assuming electrons are Maxwell-Boltzmann distributed.

$$f_e(\vec{v},\vec{r}) = n_0\left(\frac{m}{2\pi K T_e}\right)^{3/2} \exp\left[-\frac{\frac{1}{2}mv^2 - e\phi(\vec{r})}{KT_e}\right]$$

Integrating this equ. over velocity space.

$$\int_{-\infty}^{+\infty} f_e(\vec{v},\vec{r})\,d\vec{v} = n_e(\vec{r}) = \boxed{n_0 \exp\left(\frac{e\phi(r)}{KT_e}\right)}$$

linearize

Boltzmann density

$\boxed{\text{NOTE: if } x \ll 1,\ e^x \simeq 1 + x + \frac{1}{2}x^2 + \cdots}$

$$= n_0\left(1 + \frac{e\phi(\vec{r})}{KT_e}\right)$$

$$\nabla^2 \phi(\vec{r}) = \frac{e}{\epsilon_0}\left[n_0 \frac{e\phi(\vec{r})}{KT_e} - \frac{q}{e}\,\delta(\vec{r})\right]$$

In 3D spherical co-ordinate,

$$\nabla^2 \rightarrow \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \cancelto{0}{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta}} + \cancelto{0}{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \xi^2}}$$

usually $\xi \equiv \phi$, but since we are representing potential with ϕ

Since we considered $n_e \rightarrow n_e(\vec{r})$

$$\nabla_r^2 \phi(\vec{r}) = \frac{e}{\epsilon_0}\left[n_0 \frac{e\phi(\vec{r})}{KT_e} - \frac{q}{e}\delta(\vec{r})\right]$$

$$\boxed{\nabla_r^2 \phi(\vec{r}) - \frac{e^2 n_0}{\epsilon_0 K T_e}\phi(\vec{r}) = 0}$$

$$\Downarrow$$

$$\boxed{\nabla_r^2 \phi(\vec{r}) - \frac{1}{\lambda_D^2}\phi(\vec{r}) = 0}$$

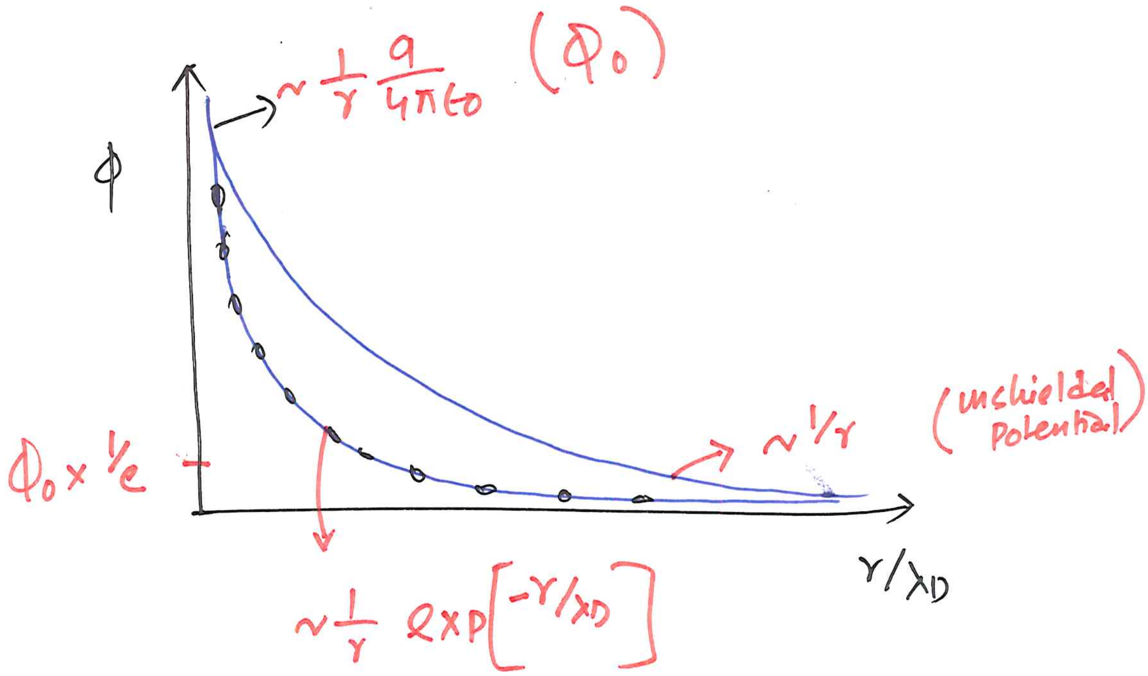
Let's consider a trial solution of the following form,

$$\phi(\vec{r}) = \phi_0 e^{-r/\lambda_D}$$

where, $\boxed{\phi_0 = \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{r}}$ Free space solution

i.e. $r \rightarrow \infty$ $\phi \rightarrow 0$

$$\boxed{\phi(r) = \underbrace{\frac{q}{4\pi\epsilon_0}}\, e^{-r/\lambda_D} \cdot \underbrace{\frac{1}{r}}}$$



if we move away from the charge

at $\lambda_D \rightarrow \phi$ decrease $1/e$ amount.

CRITERIA!

Plasma system length should be at least bigger than the Debye length. to allow shilding.

In 2D,

$$\phi(r) \doteq a_0 K_0 (r/\lambda_D)$$

Bessel function of order zero.

For 1D,

INTEGRATION CONST.

$$\boxed{\phi(x) = a_0 \exp\left(-\frac{|x|}{\lambda_D}\right)}$$

$$\frac{d^2}{dx^2}\phi(x) = \frac{e}{\epsilon_0}\left(n_e(x) - n_0 - \frac{q}{e}\delta(x)\right)$$

$$\boxed{q\,\delta(x) \rightarrow \text{slab of charge}}$$

Important:

At origin, $x = 0$, the second derivative is singular. So the δ-function is recovered in this way.

For MOBILE IONS

$n_i \neq n_0$ anymore,

$$n_i = n_0 \exp\left[-\frac{e\phi(\vec{r})}{KT_i}\right] \rightarrow$$ Only when ions are Boltzmann Distributed

$$\simeq n_0 \left(1 - \frac{e\phi(\vec{r})}{KT_i}\right)$$

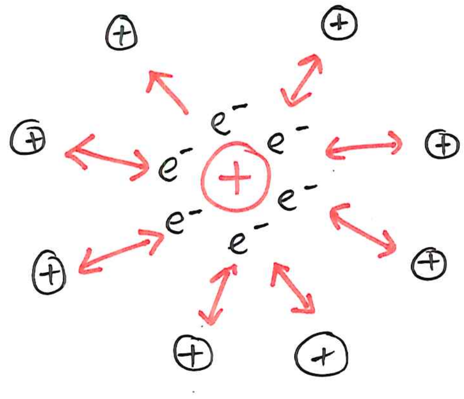
Assuming singly charged ions, the final form of potential,

$$\boxed{\phi(r) = \frac{q}{4\pi\epsilon_0} \exp\left[-r/\lambda_{Def}\right] \frac{1}{r}}$$

where,

$$\boxed{\frac{1}{\lambda_{Def}^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2}}$$

This only works when both the ion and electrons have time to respond.



For immobile ions case

$\leftrightarrow$ ✗

For mobile ions

$\leftrightarrow$ ✓

NET CHARGE IN DEBYE SPHERE

$$n_e(r) - n_0 = n_0\left(1 + \frac{e\phi(r)}{kT_e}\right) - n_0$$

$$= n_0 \frac{e\phi(r)}{kT_e}$$

$$= n_0 \times \frac{e}{kT_e} \times \phi_0 e^{-r/\lambda_D}$$

$$= n_0 \times \frac{e}{kT_e} \times \frac{q}{4\pi\epsilon_0} \times \frac{1}{r} e^{-r/\lambda_D}$$

$$= \frac{1}{e} \times \frac{n_0 e^2}{\epsilon_0 kT_e} \times \frac{q}{4\pi} \times \frac{1}{r} e^{-r/\lambda_D}$$

$$= \frac{q}{e} \times \frac{1}{\lambda_D^2} \times \frac{1}{4\pi} \times \frac{1}{r} e^{-r/\lambda_D}$$

$$= \frac{q}{e} \times \frac{1}{4\pi \lambda_D^2} \times \frac{1}{\lambda_D} \times \frac{\lambda_D}{r} \, e^{-r/\lambda_D}$$

$$\boxed{= \frac{q}{e} \, \frac{1}{4\pi \lambda_D 3} \, \frac{e^{-r/\lambda_D}}{r/\lambda_D}}$$

Finally' integrating ~~over~~ net charge over space,

e>0

$$-e \int_0^{2\pi} \int_0^{\pi} \int_0^{\alpha} [n_e(r) - n_0] \, r^2 \underbrace{\sin\theta \, dr \, d\theta \, d\xi}_{4\pi}$$

$$= -q \int_0^{\alpha} r \exp[-r] \, dr$$

$$= -q$$

The amount of charge needed to shield is -q for a charg +q.

For, $T_i \neq T_e$

$$\boxed{= -q \frac{T_e}{T_e + T_i}}$$

For electron.

$$\boxed{= q \frac{T_i}{T_e + T_i}}$$

For ions

PLASMA PARAMETER

$$N_p = n \lambda_D^3$$

n → number density

λ_D^3 → volume.

number of particles in a Debye sphere / cube

~~when N_p increases~~

when, density n increases

N_p decreases. because $\lambda_D \simeq \sqrt{\frac{1}{n}}$

$$N_p : \frac{\text{Debye length}}{\text{interparticle separation} \sim n^{-1/3}}$$

N_p is large : average separation is much small compared to λ_D

PLASMA PARAMETER ONLY MAKE SENSE IN 3D.