L05: Basic Plasma Parameters

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A Short Recap of Plasma

Plasma is

- a collection of free charged particles and neutrals moving in random directions.
- on the average, electrically neutral or quasi-neutral.

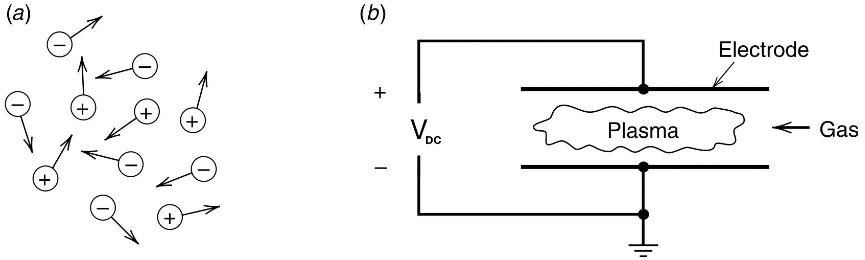


Figure 1: Schematic view of (a) a plasma and (b) a discharge. (Lieberman and Lichtenberg)

Discharge Process

A plasma discharge consists of a voltage source that drives current through a low-pressure gas between two parallel conducting plates or electrodes.

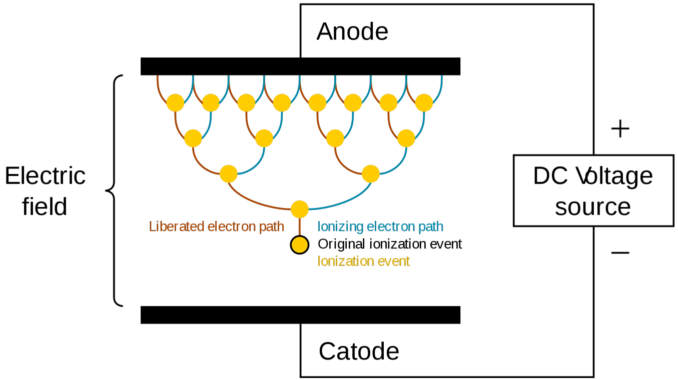


Figure 2: Discharge process in a nutshell. (D. Fadeev, Wiki)

Plasmas are Weakly Coupled

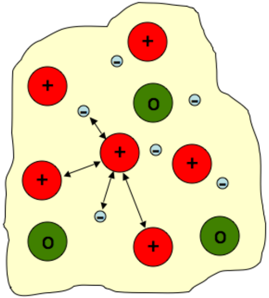


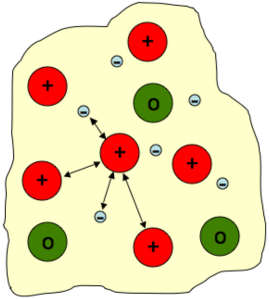
Figure 3: Representation of plasma as a collection of ions (+), electrons (-) and neutral atoms (o). (Particle In Cell Consulting LLC)

- Electron density $n \approx n_e \approx n_i$
- Typical inter-particle spacing $L \sim n^{1/3}$ (Consider, a box of size L^3 contains 1 particle on average, $L^3 n = 1$).

Interaction between nearest neighbors is typically weak

$$\frac{\textit{Typical Potential Energy of nearest neighbors}}{\textit{Typical Kinetic Energy of nearest neighbors}} \ll 1$$

Plasmas are Weakly Coupled



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Interaction between nearest neighbors is typically weak

$$\frac{\textit{Typical Potential Energy of nearest neighbors}}{\textit{Typical Kinetic Energy of nearest neighbors}} \ll 1$$

$$\frac{P.E.}{K.E.} \sim \frac{e^2/L}{\frac{1}{2}mv^2} \approx \frac{e^2 n^{1/3}}{T} \ll 1$$

NOTE: In plasma physics, temperatures are measured in energy units, so Boltzmann's constant $k_B = 1$.

To be a plasma, must be sufficiently hot and/or sufficiently rarefied.

Plasmas are Weakly Coupled

Parameters	Magnetic Fusion (MF)	Solar Wind	Galactic Center
$n_e(cm^{-3})$	10^{14}	10	10^2
T_e	10 keV	10 eV	2 keV
$\frac{P.E.}{K.E.} \sim \frac{e^2 n^{1/3}}{T}$	10^{-6}	10^{-8}	10^{-10}

Table 1: Parametric comparison in standard plasma environments

Basic Plasma Parameters: Plasma Frequency

1. Time Scale: Plasma period ($\tau_{i,e}$)

Characteristic time scale for plasma

$$\tau_{i,e} = \frac{2\pi}{\omega_{pi,e}}, \ where\ \omega_{pi,e} = \sqrt{\frac{e^2 n_{i,e}}{\epsilon_0 m_{i,e}}}$$

$where, n = density,\ e = elementary\ charge,\ m = mass$

Consider a plasma slab (no walls). Displace all electrons to the right a small distance x_{e0}, and release them

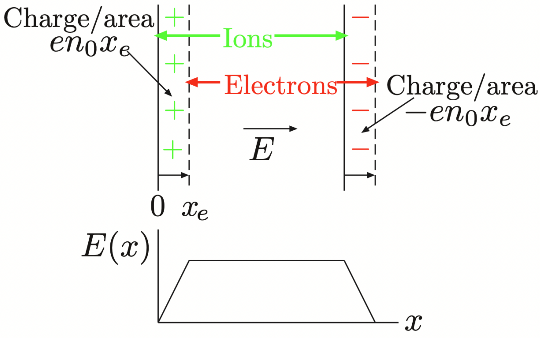

Figure 4: Representation of plasma oscillation. (M. Lieberman, UCB)

Basic Plasma Parameters: Plasma Frequency

- Maxwell's equations (parallel plate capacitor)

$$E = \frac{en_0 x_e(t)}{\epsilon_0}$$

- Newton's law (electron motion)

$$m\frac{d^2x_e(t)}{dt^2} = -eE = -\frac{e^2 n_0}{\epsilon_0} x_e(t)$$

- The equation takes the following form

$$\frac{d^2x_e(t)}{dt^2} + \left(\frac{e^2 n_0}{\epsilon_0 m}\right) x_e(t) = 0$$

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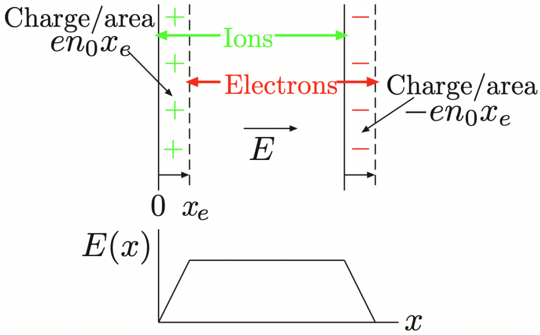

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Equation for harmonic oscillator

$$\ddot{x} + \omega_{pe}^2 x = 0$$

which has a solution of the following form,

$$x \sim e^{-i\omega_{pe} t}$$

Electron Plasma Frequency (ω_{pe})

$$\omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$$

$where, n = electron\ density,\ e = elementary\ charge,\ m = electron\ mass$

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Practical formula

$$\frac{1}{2\pi}\omega_{pe} = f_{pe}(\text{Hz}) = 9000\sqrt{n_0}$$

where n_0 is electron density in cm^{-3}

Basic Plasma Parameters: Plasma Frequency

	Magnetic Fusion (MF)	Ionosphere	Solar Wind	Galactic Center
$\omega_{pe}(s^{-1})$	6×10^{11}	6.3×10^{4}	2×10^{5}	6×10^{5}

Table 2: Typical plasma frequencies in standard plasma environments

Basic Plasma Parameters: Debye Length

2. Debye Length (λ_D)

Characteristic length for plasma

$$\lambda_D = \sqrt{\frac{\epsilon_0 kT}{e^2 n}}$$

NOTE: For electron: λ_{De}, and for ion: λ_{Di}, where n represents the respective species.

- The characteristic velocity in plasma,

$$\text{Thermal Velocity}(V_{thi,e}) = \sqrt{\frac{kT_{i,e}}{m_{i,e}}}$$

- The characteristic length (λ_D),

$$\lambda_D = V_{th}/\omega_p$$

$$\lambda_D = \sqrt{\frac{kT}{m}}\sqrt{\frac{\epsilon_0 m}{e^2 n}} = \sqrt{\frac{\epsilon_0 kT}{e^2 n}}$$

Basic Plasma Parameters: Debye Length

Debye Shielding

Using the characteristic length for plasma, one can derive the expression for the shielded potential,

- In 3D (spherical)

$$\phi(r) = \frac{q}{4\pi\epsilon_0} \exp\left[-\frac{r}{\lambda_D}\right]\frac{1}{r}$$

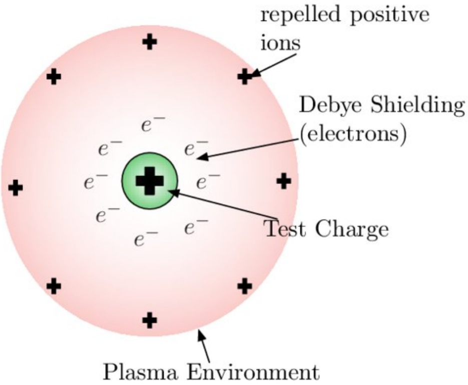

Figure 4: Debye Shielding (Hanspeter Schaub, UC, Boulder).

Basic Plasma Parameters: Debye Length

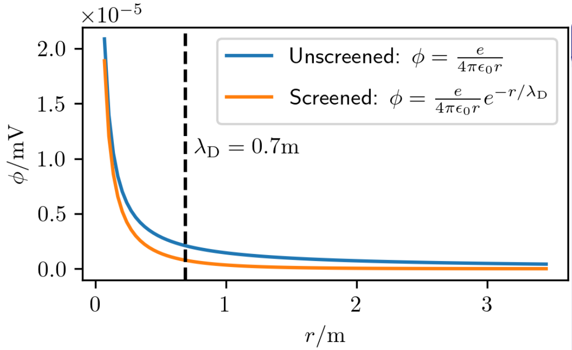

Figure 5: Estimation of Debye Shielding (scipython.com)

Python script

```
#> python debye_shielding.py -Te 1e4 -den 1e14
```

Te = Electron temperature in eV (default value: 1.e8)

den = Electron density in (cm^{-3}) (default value: 1.e26)

For help:

```
#> python debye_shielding.py -h
```

GitHub Page

Basic Plasma Parameters: Debye Length

- Higher temperature (K.E.) of electrons $\rightarrow$ larger Debye length
- Higher density $\rightarrow$ smaller Debye length

	Magnetic Fusion (MF)	Ionosphere	Solar Wind	Galactic Center
λ_D (cm)	7×10^{-3}	20	7×10^{2}	3×10^{3}

Table 3: Typical plasma Debye length in standard plasma environments

Basic Plasma Parameters: Plasma Parameter

Plasma Parameter

$$N_p = n\lambda_D^3$$

n = plasma density
λ_D^3 = Volume of Debye Cube

- Higher density $\rightarrow$ smaller N_p ($\lambda_D \sim \sqrt{\frac{1}{n}}$)
- N_p : $\frac{\textit{Debye Length}}{\textit{Interparticle Separation} \sim n^{-1/3}}$
- When N_p is large $\rightarrow$ average separation is much smaller compared to λ_D

Basic Plasma Parameters: Plasma Parameter

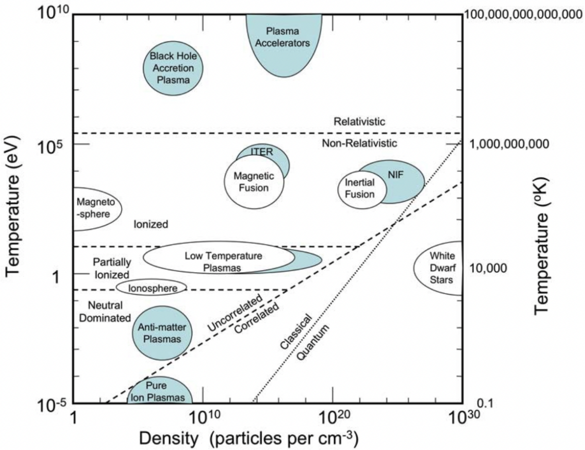

Figure 6: Wide range of possible plasma parameters. Plasmas above the line marked "Uncorrelated-Correlated" correspond to $N_p \gg 1$ (Plasma Science (2007), NRC)

Thank you