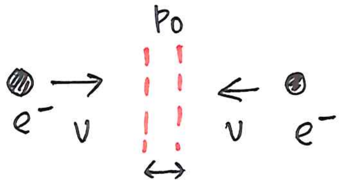
COLLISIONS IN PLASMAS (UNMAGNETIZED)

1. HEAD ON COLLISION



CLOSEST DISTANCE TWO CHARGE PARTICLE CAN APPROACH.

- Two electrons moving in opposite direction. heading towards each other.
- each have K.E. : $\frac{1}{2}mv^2$
- the interaction force between them : F_c (Coulomb interaction) which will slow them down.

At the closest approach (p_0)

$$K.E._{(TOT)} \longrightarrow \text{Interaction potential energy.}$$

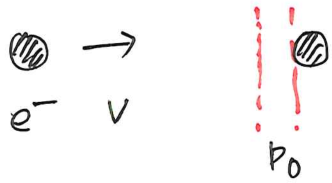
$$2 \times K.E = mv^2 = F_c = \frac{e^2}{4\pi\epsilon_0 p_0}$$

$$\Rightarrow \boxed{mv^2 = \frac{e^2}{4\pi\epsilon_0 p_0}}$$

For head on collision with two moving electron the closest approach

$$\boxed{p_0 = \frac{e^2}{4\pi\epsilon_0 m v^2}} \quad --- \; (1)$$

1.A. IF A MOVING CHARGE COLLIDES WITH A STATONARY CHARGE OF SAME KIND (MAG)



Similarly,

$$K.E_{(TOT)} = F_c$$

$$\Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} \frac{e^2}{2\pi \epsilon_0 p_0}$$

$$\boxed{\Rightarrow m v^2 = \frac{e^2}{2\pi \epsilon_0 p_0}}$$

The closest approach in this case,

$$\boxed{p_0 = \frac{e^2}{2\pi \epsilon_0 m v^2}} \quad --- \text{(2)}$$

Now, having a collection of charge particle, the most probable velocity would be the thermal velocity.

$$\frac{1}{2} m v^2 \approx \frac{1}{2} KT$$

$$\boxed{\Rightarrow \; m v^2 \approx KT}$$

So approximately, eqn. 1 can be written as,

$$\boxed{\lambda_D = \sqrt{\frac{\epsilon_0 KT}{e^2 n}}}$$

$$\boxed{N_P = n \lambda_D^3}$$

$$p_0 = \frac{e^2}{4\pi\epsilon_0 KT}$$

$$= \frac{1}{4\pi} \times \frac{e^2 n}{\epsilon_0 KT} \times \frac{1}{n}$$

$$= \frac{1}{4\pi} \frac{1}{\lambda_D^2} \times \frac{\lambda_D^3}{N_P}$$

$$\boxed{p_0 = \frac{\lambda_D}{4\pi N_P}} \ll \lambda_D$$

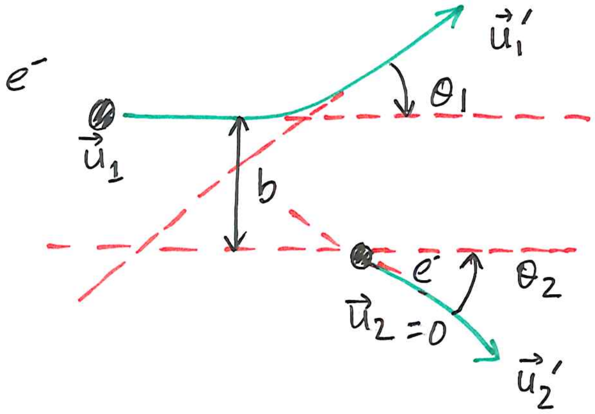
Since, $N_P \gg 1$

So, the closest approach (p_0) is much smaller than the debye length.

Hence, the debye sphere is uniformly filled with electrons.

- If $N_p \leq 1$, one electron doesnot gets closer to another in a debye sphere. As a result debye sphere will end up like voids.

2. GLANCING COLLISION:



$\boxed{b \rightarrow \text{impact parameter}}$

The movement of the particle 2 can either be considered stationary or otherwise based on the frame of reference.

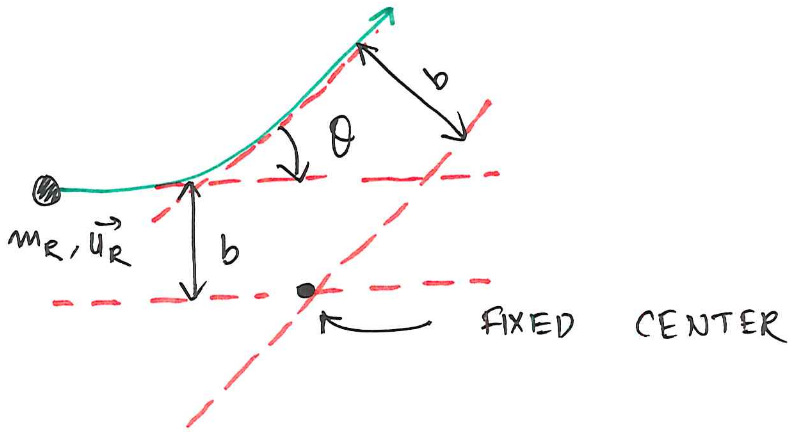
CENTER OF MASS COORDINATE

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \qquad \text{position}$$

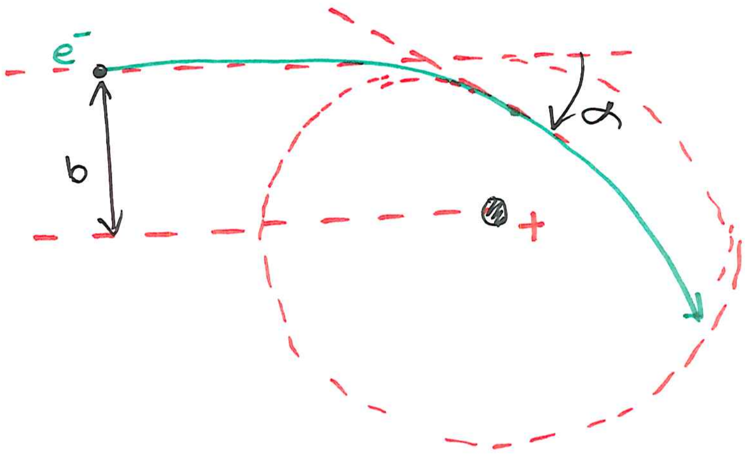
$$\vec{U} = \frac{m_1 \vec{u}_1 + m_2 \vec{u}_2}{m_1 + m_2} \qquad \text{velocity}$$

$$m_R = \frac{m_1 m_2}{m_1 + m_2} \qquad \text{mass (reduced)}$$

Collision in CENTER OF MASS



ELECTRON - ION COLLISION:



- collision of electron with ion will change both the direction and magnitude of electon velocity.

- The electron will only start interacting with ions once they enter the sphere of influence.

- The electron will only interact for a limited time.

The time of interaction (τ)

$$\tau \approx \frac{b}{v} = \frac{\text{impact parameter}}{\text{impact velocity}}$$

Now, the change in electrons momentum,

$$\Delta(mv) = |F_c \cdot \tau| \approx \frac{e^2}{4\pi\epsilon_0 b v}$$

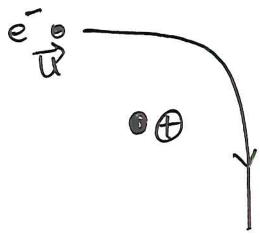
impact param.

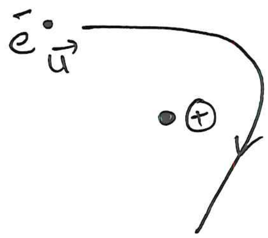
(Coulomb Force) $F_c = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$

For collision $r \sim b$

LARGE ANGE SCATTERING

WHEN $\alpha \geq 90^\circ$





For $\alpha \simeq 90^\circ$

$$\Delta(mv) \approx mv$$

$$\approx \frac{e^2}{4\pi\epsilon_0 p_{90} u}$$

$$\boxed{\Rightarrow p_{90} = \frac{e^2}{4\pi\epsilon_0 m u^2}}$$

Now, for HEAD ON

$\alpha \simeq 180^\circ$

$$\Delta(mv) = 2mv \quad \longrightarrow \quad p_{180} \approx \frac{e^2}{}$$

CROSS SECTION (COLLISIONAL)

$$\sigma_{90} = \pi p_{90}^2 \approx \frac{e^4}{16\pi \epsilon_0^2 m^2 u^4}$$

if, we consider the thermal velocity approximation

$$\frac{1}{2} m u^2 \approx \frac{1}{2} KT$$

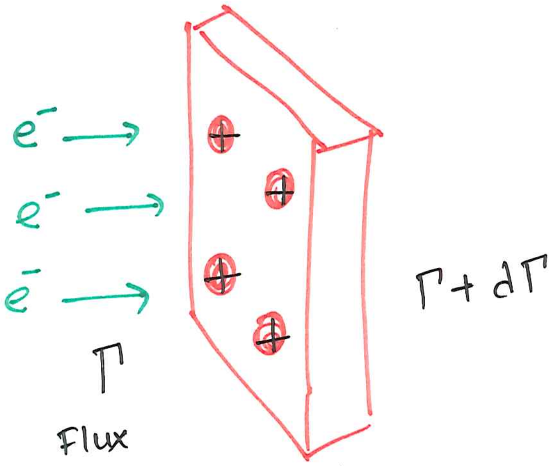
$$\sigma_{90} = \frac{e^4}{16\pi (\epsilon_0 KT)^2}$$

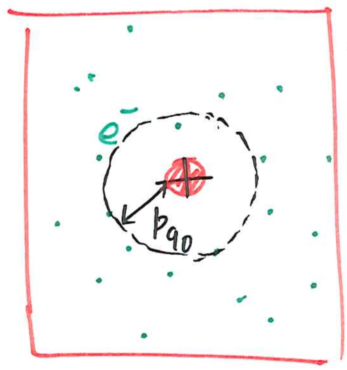
$$\boxed{\lambda_D = \sqrt{\frac{\epsilon_0 KT}{e^2 n}} \qquad N_p = n \lambda_D^3}$$

$$= \frac{1}{16\pi} \left(\frac{e^2 n}{\epsilon_0 KT}\right)^2 \times \frac{1}{n^2}$$

$$= \frac{1}{16\pi} \cdot \frac{1}{\lambda_D^4} \cdot \frac{\lambda_D^6}{N_p^2}$$

$$\boxed{\sigma_{90} = \frac{1}{16\pi} \left(\frac{\lambda_D}{N_p}\right)^2}$$

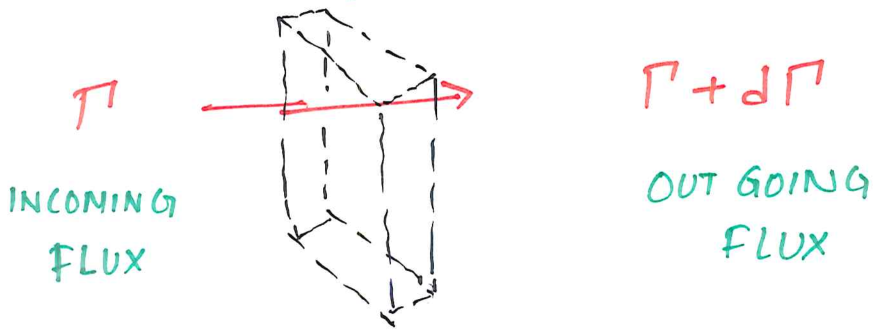




The critical impact parameter p_{90}
↓
The effective interaction radius. for the particle with given temperature.

The effective area of interaction is the collisional crosssection.

FLUX



$$\frac{\Gamma - (\Gamma + d\Gamma)}{\Gamma} = \underbrace{N}_{\text{Number of scatterer}} \cdot \underbrace{\sigma}_{\text{crosssection}} \cdot \frac{1}{\underbrace{A}_{\text{Area}}}$$

If thickness is dz

$$N = \underbrace{A}_{\text{Area.}} \cdot \underbrace{dz}_{\text{thickness}} \cdot \underbrace{n_s}_{\text{density of scatterer}}$$

$$\frac{\Gamma - (\Gamma + d\Gamma)}{\Gamma} = N \cdot \sigma \cdot \frac{1}{A} = dz \cdot n_s \cdot \sigma$$

$$\Rightarrow \quad \frac{d\Gamma}{\Gamma} = dz\, n_s\, \sigma$$

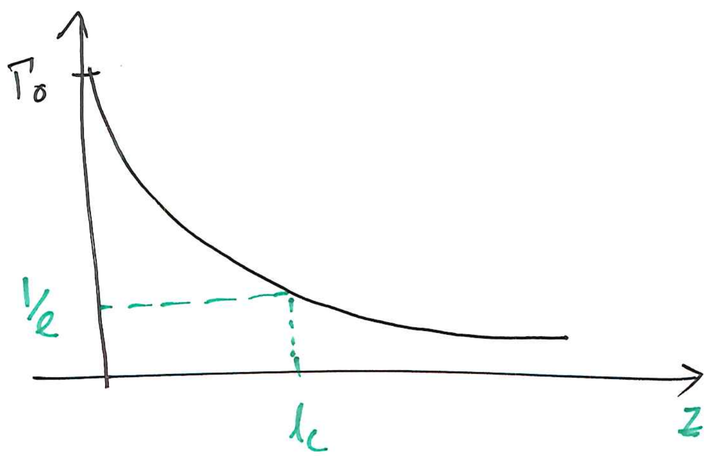
$$\boxed{\Rightarrow \Gamma = \Gamma_0\, e^{-z n_s \sigma}}$$

- Flux decays exponentially with the thickness of the slab.

$$\boxed{\Gamma = \Gamma_0\, e^{-\frac{z}{l_c}}}$$

MEAN FREE PATH

$$l_c = \frac{1}{n_s \sigma} = \frac{1}{\text{density of scatterer} \times \text{Cross section}}$$



$$\mathcal{L}_s \simeq \left(\frac{1}{n_s}\right)^{1/3}$$

– Mean distance between particles. (Also in the last class)

REVISITING

COLLISION CROSSSECTION

$$\sigma_{90} = \pi p_{90}^2 \simeq \frac{e^4}{16\pi\epsilon_0^2 m^2 u^4} = \frac{1}{16\pi}\left(\frac{\lambda_D}{N_p}\right)^2$$

MEAN FREE PATH

$$\boxed{N_p = n\lambda_D^3}$$

$$l_c = \frac{1}{n\sigma_{90}} = 16\pi\lambda_D N_p$$

COLLISION FREQUENCY

$$\nu_{ei} = n\cdot\sigma_{90}\cdot u = \underbrace{n}_{\frac{N_p}{\lambda_D^3}}\cdot\frac{1}{16\pi}\cdot\left(\frac{\lambda_D}{N_p}\right)^2\cdot\underbrace{u}_{\omega_{pe}\lambda_D}$$

$$= \frac{ne^4}{16\pi\epsilon_0^2 m^2 u^3}$$

$$\cancel{= \frac{ne^2}{\epsilon_0 m}\cdot\frac{e^2}{16\pi\epsilon_0 m u^3}}$$

$$\cancel{= \omega_{pe}^2 \cdot e^2 .}$$

$$\boxed{\Rightarrow \nu_{ei} \approx \frac{\omega_{pe}}{N_p}}$$

$$\boxed{\begin{aligned}\omega_{pe} &= \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}\\ \lambda_D &= \sqrt{\frac{\epsilon_0 KT}{e^2 n}}\end{aligned}} \longrightarrow u = \omega_{pe}\lambda_D$$

TRICK: multiply and use $mu^2 = KT$

RESISTIVITY

$$\eta = \frac{m}{ne^2}\nu_{ei} = \frac{e^2}{16\pi\epsilon_0^2 m u^3}$$

more collision $\longrightarrow$ more resistivity

$$\eta \rightarrow \nu_{ei} \rightarrow \sigma \rightarrow p \rightarrow \lambda_D, N_p$$

TO CONSIDER SMALL ANGLE COLLISION (NON-TRIVIAL)

Using SPITZER LOGARITHM we can approximate such contribution.

$$\Lambda \approx 12\pi N_p \qquad \ln\Lambda \simeq 1-10$$

small contribution.

Now, l_c becomes.

$$\boxed{l_c = \frac{1}{n\sigma_{90}\ln\Lambda}}$$

MEAN TIME BETWEEN COLLISION

$$\boxed{\tau_c \sim \frac{l_c}{v_{th}} \sim \frac{\lambda_D N_p}{\sqrt{\frac{KT}{m}}} = \frac{1}{\nu_{ei}} = \frac{N_p}{\omega_{pe}}}$$

HOW TO DEFINE A COLLISIONLESS PLASMA

- Either the system length has to be smaller than mean free path (l_c)

- Plasma measurements are smaller in time scale compared to τ_c.

For large N_p

$n^{-1/3}$

$l_{avg.} \ll \lambda_D \ll l_c$

avg. distance between scatter.

Debye length

mean free path

PLASMA RESISTIVITY

(~~[illegible]~~ MAIN CONTRIBUTOR
$e^- \longrightarrow \oplus$)

1. UNMAGNETIZED, STATIONARY CASE — ONLY MOVING ELECTRON., IONS STATIONARY
2. 1D
3. IF MAG. FIELD, IT'S ASSUMED TO BE PARALLEL TO ELECTRON MOVEMENT

CURRENT (J) : $\vec{j} = -en\vec{u}$

$$\Rightarrow j_x = -en u_x$$

MOMENTUM EQN. : $m\dfrac{d\vec{u}}{dt} = q\vec{E}$

$$\Rightarrow m\frac{du_x}{dt} = qE_x$$

Because of the resistance, we assume some momentum will be lost per unit time. This loss is counted by adding to the momentum equation.

$$\boxed{m\frac{du_x}{dt} = qE_x - mu\nu_c}$$

$\nu_c = \dfrac{1}{\tau_c}$

For steady state:

$$0 = qE - mu\nu_c$$

$$\Rightarrow \quad qE = mu\nu_c = mu\frac{\omega_{pe}}{N_p}$$

$$\boxed{\Rightarrow \quad u = -\frac{eN_p}{m\omega_{pe}}E}$$

For electrons
$q = -e$

So, current,

$$j = -enu$$

$$= \frac{e^2 n}{m\omega_{pe}} N_p E$$

$$\boxed{j = \frac{e^2 n}{m\nu_c} E}$$

$\rightarrow \underline{\sigma}$ conductivity.

$$\boxed{\underline{\sigma} = \frac{e^2 n}{m\nu_c} = \frac{\omega_{pe}^2 \epsilon_0}{\nu_c}}$$

$$\boxed{\nu_c \approx \frac{\omega_p}{N_p}}$$

$$\boxed{\underline{\sigma} \approx N_p \omega_{pe} \epsilon_0}$$

CONDUCTIVITY

RESISTIVITY

$$\boxed{\xi = \frac{1}{\underline{\sigma}} = \frac{1}{N_p \omega_{pe} \epsilon_0}}$$

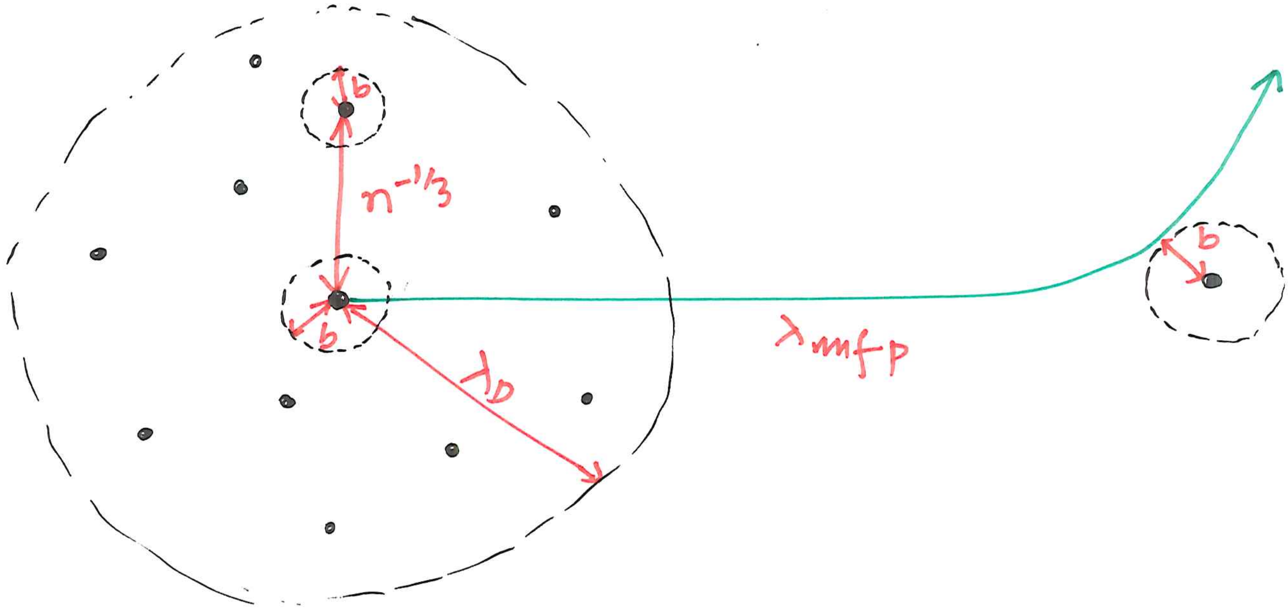
CONSIDERING SMALL ANGLE SCATTERING

$$\boxed{\xi = \frac{e^2\sqrt{m}}{16\pi\epsilon_0 (KT)^{3/2}} \cdot \ln\Lambda}$$

$\Lambda = 12\pi N_p$

While this tells that RESISTIVITY does not depend on density, in actual, adding SPITZER LOGARITHM we can see it actually depends upon density. weakly.

REVIEW OF FUNDAMENTAL LENGTH SCALE IN PLASMA



Distance of closest approach for 90° scatter	:	Avg. Interparticle spacing	:	Debye shielding length	:	Avg. Mean Free path
b	:	$n^{1/3}$	:	λ_D	:	λ_{mfp}
N_p^{-1}	:	$N_p^{-1/3}$	:	1	:	$\frac{N_p}{\ln N_p}$
10^{-6}	:	10^{-2}	:	1	:	10^{5}