COLLISION IN MAGNETIC FIELD

CHARACTERISTIC GYRO RADIUS $\quad r_L = \dfrac{mv}{qB}$

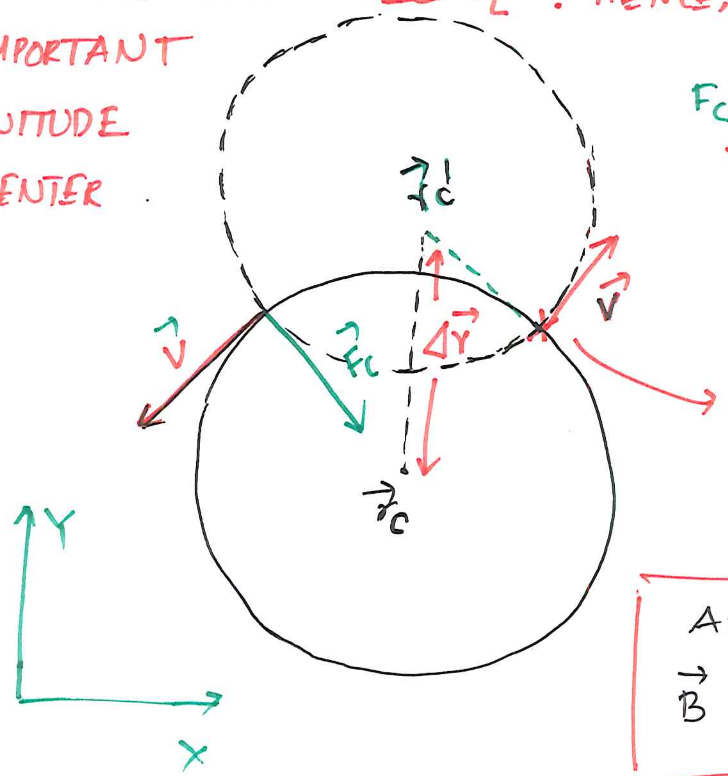
GYRO FREQUENCY $\quad \Omega_c = \dfrac{qB}{m}$

GYRO PERIOD $\quad T_c = \dfrac{2\pi}{\Omega_c}$

CONSIDER A SITUATION WHERE A CHARGE PARTICLE COLLIDES WITH ANOTHER UNDER THE INFLUENCE OF MAGNETIC FIELD

DUE TO COLLISION, THE MAXIMUM DISPLACEMENT OF GYRO CENTER WILL r_L. HENCE, r_L WILL REMAIN AN IMPORTANT ORDER OF MAGNITUDE FOR GUIDING CENTER DISPLACEMENT.

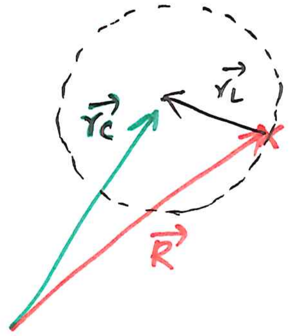
$F_c = q\vec{v} \times \vec{B}$ → LORENTZ FORCE



Assume, $\vec{B} = B\hat{z}$

$\Delta\vec{r}$ ⟶ Change in Gyrocenter

TWO PARTICLES:



unit vector direction in $\vec{F_c}$

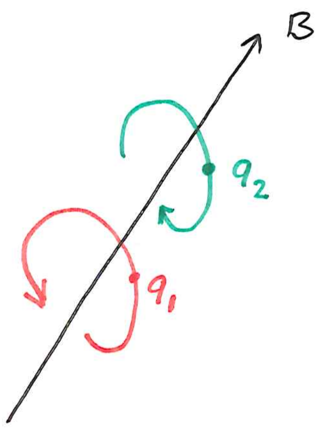
First particle.
$$\vec{r}_c^{\,(1)} = \vec{R}^{(1)} + r_L \cdot \hat{a}$$

$$= \vec{R}^{(1)} + \frac{m_1 v_1}{q_1 B}\, \hat{v} \times \hat{B} = \vec{R}^{(1)} + \frac{m_1}{q_1 B} \left(v_1 \hat{v}_1\right) \times \left(\frac{\vec{B}}{B}\right)$$

$$= \vec{R}^{(1)} + \frac{m_1}{q_1 B^2}\, \vec{V}^{(1)} \times \vec{B}$$

Second particle.
$$\vec{r}_c^{\,(2)} = \vec{R}^{(2)} + \frac{m_2}{q_2 B^2}\, \vec{V}^{(2)} \times \vec{B}$$

WHEN THE COLLISION IS IMMINENT



- Rotating in the opposite direction.
- Different velocity along parallel direction.

they only collide once.

AFTER THE COLLISION (ELASTIC)

The particles do not change positions,

$$\boxed{\vec{R}^{(1)} \simeq \vec{R}^{(2)} \Rightarrow \Delta R^{(1,2)} = 0}$$

only the velocity and magnitude.

The interaction will be very short as well.

So, the change in gyro center

$$\boxed{\begin{aligned}\Delta \vec{r}_c^{\,(1)} &= \frac{m_1}{q_1 B^2}\, \Delta \vec{V}^{(1)} \times \vec{B} \\ \Delta \vec{r}_c^{\,(2)} &= \frac{m_2}{q_2 B^2}\, \Delta \vec{V}^{(2)} \times \vec{B}\end{aligned}}$$

NOTE: After the collision, the position does not change but the gyro center does.

CONSERVATON OF MOMENTUM

$$m_1 \Delta \vec{u}^{(1)} + m_2 \Delta \vec{V}^{(2)} = 0$$

$$q_1 \Delta \vec{r}_c^{\,(1)} + q_2 \Delta \vec{r}_c^{\,(2)} = 0$$

CASES-I (BOTH ELECTRONS)

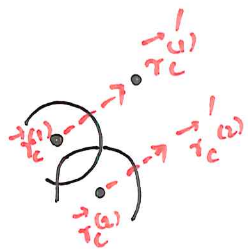
When, $q_1 = q_2$, $\Rightarrow \Delta \vec{r}_c^{\,(1)} + \Delta \vec{r}_c^{\,(2)} = 0 \Rightarrow \Delta \vec{r}_c^{\,(1)} = -\Delta \vec{r}_c^{\,(2)}$

1. CHANGE IN POSITION OF THE GYROCENTERS ARE CONSTANT (AVERAGE)
2. JUMPS OF PARTICLES AFTER COLLISION ARE SAME BUT IN OTHER DIRECTION.

CASE-II (ELECTRON AND SINGLY CHARGED POSITIVE ION)

When, $q_1 = -q_2$, $\Rightarrow \Delta \vec{r}_c^{\,(1)} - \Delta \vec{r}_c^{\,(2)} = 0$

$$\Rightarrow \Delta \vec{r}_c^{\,(1)} = \Delta \vec{r}_c^{\,(2)}$$



We know from before that,

the maximum jump of gyro center can be ~~[illegible]~~ r_L and the order of magnitude of guiding center displacement.

→ Gyro radius depends on mass

→ Smaller gyro center will be controlling the movement.
$r_{Le} << r_{Li}$

Hence,
electrons will control the movement of ions.

SIMILAR TO MOMENTUM, ENERGY IS ALSO CONSERVED IN SUCH COLLISIONS.

RESISTIVITY BY NEUTRAL COLLISION

Resistivity due to neutral collision plays a big role in the ionosphere. In particular, the lower ionosphere (E & F region), neutral collision dominates the plasma conductivity.

MOMENTUM EQN. IN WEAKLY COLLISIONAL PLASMA

$$m \frac{d}{dt}\vec{u} = \underbrace{q\vec{E}}_{\text{E-field}} + \underbrace{q\vec{u}\times\vec{B}}_{\text{B-field}} - \underbrace{m\nu\vec{u}}_{\text{Collision}}$$

NOTE: At this point this equ. represent a single particle motion instead of the whole species.

$\nu \rightarrow$ Collision freq. (const)

$\lambda_c \rightarrow \frac{u}{\nu}$ mean free path, $\lambda_c \propto u$

i.e. Conductivity $\sigma \downarrow$ decrease with increasing $\nu \uparrow$

IN A STEADY STATE SITUATION:

$$0 = q\vec{E} + q\vec{u}\times\vec{B} - m\nu\vec{u} \quad \text{--- } (1)$$

TO FIND THE PARALLEL AND PERPENDICULAR COMPONENT OF THE VELOCITY (W.R.T. $\vec{B}$), WE TAKE CROSS PRODUCT WITH $\vec{B}$.

$$\boxed{0 = q\vec{E} + q\vec{u}\times\vec{B} - m\nu\vec{u}} \quad \times \; \vec{B}$$

$$\Rightarrow \; 0 = q(\vec{E}\times\vec{B}) + \underbrace{q\,(\vec{u}\times\vec{B})\times\vec{B}}_{-q\,\vec{u}_\perp B^2} - m\nu(\vec{u}\times\vec{B})$$

NOTE: USE

$(\vec{A}\times\vec{B})\times\vec{C}$

$= \vec{A}\times(\vec{B}\times\vec{C}) - \vec{B}\times(\vec{A}\times\vec{C})$

$$\Rightarrow \; 0 = q(\vec{E}\times\vec{B}) - q\,u_\perp B^2 + \boxed{\nu\,\frac{m}{q}\left(qE_\perp - m\nu\,\vec{u}_\perp\right)} \quad \text{--- } (2)$$

COMPONENTS:

$\boxed{\parallel \vec{B}}$ $$u_\parallel = E_\parallel\, q\, \frac{1}{m\nu}$$ From (1)

$\boxed{\perp \vec{B}}$ $$u_\perp = \frac{q\vec{E}\times\vec{B} + m\nu E_\perp}{qB^2 + m^2\nu^2\frac{1}{q}}$$ From (2)

$$= \left(\frac{q}{m}\right)^2 \frac{\vec{E}\times\vec{B} + (m/q)\,\nu\,\vec{E}_\perp}{\Omega_c^2 + \nu^2}$$

NOTE: We need a non zero collision frequency in order to have a steady state finite current due to external electric field.

THE PERPENDICULAR COMPONENT CAN BE REPRESENTED AS,

$$\Rightarrow \quad u_\perp = \beta \frac{\vec{E} \times \vec{B}}{B^2} + \alpha \frac{E_\perp}{B}$$

where,

$$\boxed{\alpha \equiv \frac{\nu/\Omega_c}{1 + \nu^2/\Omega_c^2} \qquad \beta \equiv \frac{1}{1 + \nu^2/\Omega_c^2}}$$

$$\alpha = \left(-\tfrac{1}{2}, \tfrac{1}{2}\right), \quad \beta = (0, 1)$$

$$0 \leq |\alpha| \leq 1/2 \qquad 0 < \beta < 1$$

For ions

$$\text{Now, } \frac{\Omega_{ci}}{\nu_i} \simeq \frac{eB}{M_i} \cdot l_{ci} \sqrt{\frac{m_i}{KT_i}} \rightarrow \frac{1}{4}$$

(Thermal velocity)

For electrons

$$\frac{\omega_{ci}}{\nu_e} = \frac{eB}{m_e} \, l_c \sqrt{\frac{m_e}{KT_e}}$$

GENERAL CURRENT

$$\vec{J} = e\left[n_i \vec{u}_i - n_e \vec{u}_e\right]$$

Assuming quasi-neutrality, $n = n_i = n_e$

PARALLEL TO B

$$\vec{J}_{\parallel} = \sigma_{\parallel} E_{\parallel}$$

PERP. TO B

$$\vec{J}_{\perp} = \sigma_P \underbrace{\vec{E}_{\perp}}_{\text{Perpendicular to } \vec{E}} + \sigma_H \underbrace{(\vec{E} \times \hat{b})}_{\text{Perpendicular to both } \vec{E} \text{ and } \vec{B}}$$

$$\hat{b} = \frac{\vec{B}}{B}$$

$$u_{\perp} = \underbrace{\beta \frac{\vec{E} \times \vec{B}}{B^2}}_{\sim \sigma_H} + \underbrace{\alpha \frac{E_{\perp}}{B}}_{\sim \sigma_P}$$

$$\boxed{\sigma_P = \frac{en}{B}(\alpha_i - \alpha_e)}$$ PEDERSEN CONDUCTIVITY

$$\boxed{\sigma_H = \frac{en}{B}(\beta_i - \beta_e)}$$ HALL CONDUCTIVITY

$$\sigma_{\parallel} = e^2 n \left[\frac{1}{m_i \nu_i} + \frac{1}{m_e \nu_e}\right]$$ Parallel conductivity

$$\boxed{\sigma_{\parallel} = \frac{e^2 n}{m_e \nu_e}}$$ $\because m_e \ll m_i$

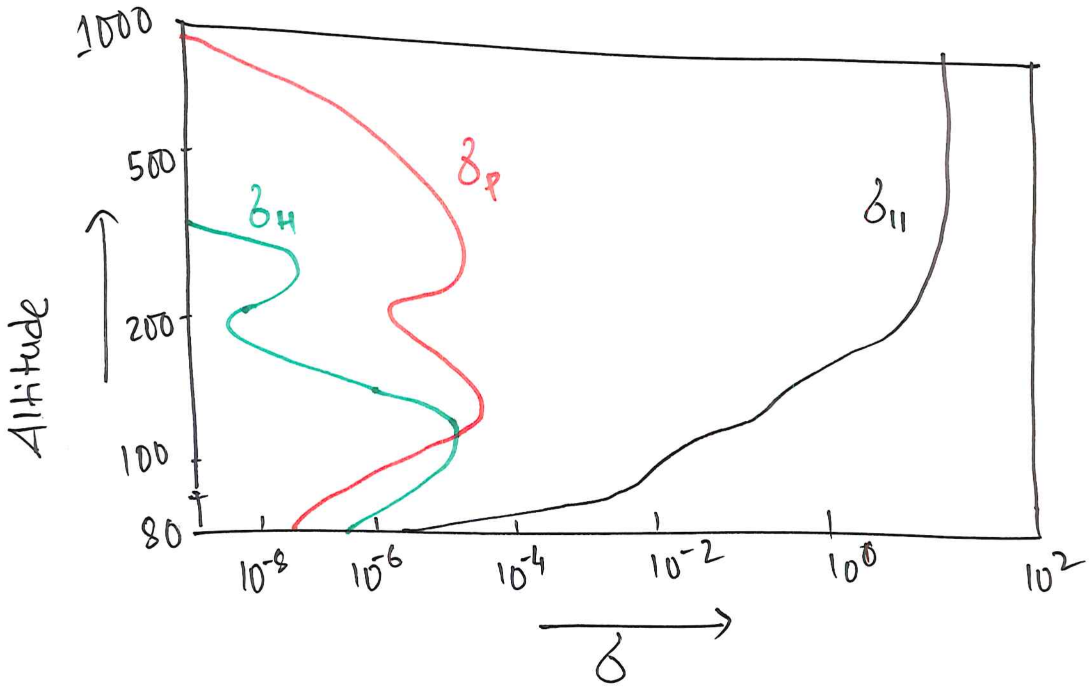
So, the general current expression becomes,

$$\vec{J} = \underset{=}{\sigma} \cdot \vec{E}, \quad \vec{B} = B\hat{z}, \quad \underset{=}{\sigma} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix}$$

conductivity tensor

In the Ionosphere,

Plasma is ionized above 60-70 Km. and significantly ionized above 90 Km.



Since, the electron density ↑ with altitude, $\sigma_{\parallel}$ ↑

For σ_P and σ_H, they highly depend on ν which ↓ as we move up in the atmosphere.

For some altitude, $\Omega_{ci} \approx \nu_i$, provided $T_i \sim T_e$

We can write, $$\frac{\omega_{ce}}{\nu_e} \approx \sqrt{\frac{m_i}{m_e}} \sqrt{\frac{T_i}{T_e}} \gg 1$$

For high altitudes,

≥ 150 km, both $\beta_e \approx \beta_i \approx 1$ i.e. $\sigma_H \approx 0$

Similarly, low altitudes

≤ 60 km. both $\beta_e \approx \beta_i \approx 0$ i.e. $\sigma_H \approx 0$

NOTE!

In general $\sigma_{\parallel}$ is much larger than σ_H and σ_P.

FACT

In polar region, where magnetic field is perpendicular to the ground, the direction of the current in the ionesphere can change with the altitude. At high altitude, $\sigma_P \uparrow\downarrow \sigma_H \downarrow\uparrow$ the ~~[illegible]~~ conductivity contribution will lead to the current flow in the same direction either $\vec{E}_\perp$ or $\vec{E} \times \hat{b}$