

EQN OF STATE

$$p = \underbrace{n}_{\frac{N}{V}} kT$$

$n =$ number of particles per unit volume
(number density)

$$\Rightarrow p = f(p, T) \rightarrow \begin{array}{l} \text{Assuming} \\ \text{Isothermal} \end{array} \quad \begin{array}{l} p \propto n \\ p = k_B T \cdot n \end{array}$$

For adiabatic
 $p \rho^{-\gamma} = \text{const.}$
 $\gamma = \frac{C_p}{C_v}$

This is the third and last relation to close the fluid representation of the system.

$$\frac{\partial T}{\partial t} = \overset{\substack{\text{Diff. coeff.} \\ \downarrow}}{D_T} \nabla^2 T \quad \leftarrow \begin{array}{l} \text{diffusion eqn.} \\ \text{for heat} \end{array}$$

$$T v_T = -D_T \nabla T \quad \leftarrow \text{heat flux}$$

Characteristic velocity for temperature propagation.

L - length scale

τ - time scale

$$v \sim \frac{L}{\tau}$$

$$v_T = -D_T \frac{\nabla T}{T}$$

$$\approx -D_T \frac{1}{L}$$

$v_T \gg u$ - smooth variation

$v_T \ll u$ - adiabatic

HEAT CAPACITIES

$$\gamma = \frac{C_p}{C_v}$$

$$C = \frac{Q}{\Delta T} = \frac{\Delta U - W}{\Delta T}, \quad W = -P\Delta V$$

internal energy (pointing to ΔU)
Work (pointing to $-W$)
change in temp. (pointing to ΔT)

Specific heat capacities:

$$C_v = \left(\frac{\Delta U}{\Delta T} \right)_v \rightarrow \left(\frac{\partial U}{\partial T} \right)_{v=\text{const}}$$

$$C_p = \left(\frac{\Delta U - (-P\Delta V)}{\Delta T} \right)_p \Rightarrow \left(\frac{\partial U}{\partial T} \right)_p + P \left(\frac{\partial V}{\partial T} \right)_p$$

For ideal gas:

$$\frac{C_p}{C_v} = \frac{5}{3}$$

$$C_v = \frac{3}{2} nK$$

$$C_p = C_v + nK$$